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Math Expressions

Making the Case for the Next-Generation NSF Program *Math Expressions*

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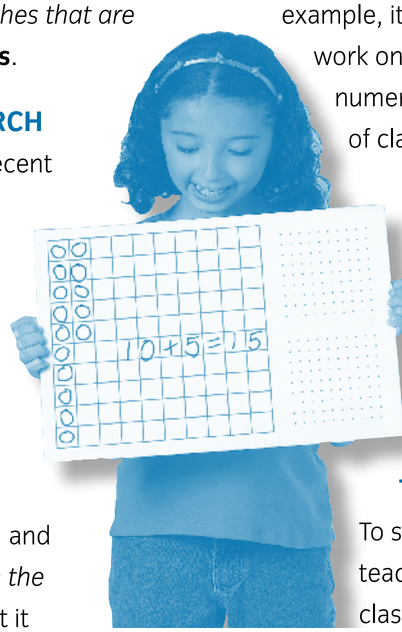
Math Expressions *author Dr. Karen Fuson taught math for three years in Chicago. Her experiences there encouraged her studies in how to effectively teach students math from an early age, begun after she obtained her Ph.D. in mathematics education from the University of Chicago. The results from her years of research, and from the research of others, helped create the approaches that are incorporated into* **Math Expressions**.

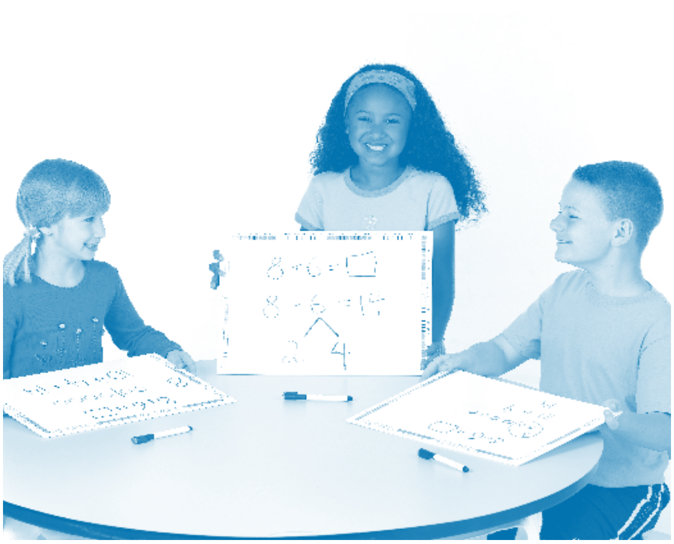
REFLECTING NATIONAL RESEARCH

Math Expressions uses the most recent research-based approaches to teaching and learning math that are summarized in the second 2000 NCTM Standards document and in two national reports from the National Research Council. The NCTM *Principles and Standards for School Mathematics, Adding It Up: Helping Children Learn Mathematics,* and *How Students Learn: Mathematics in the Classroom* summarize how important it is to base teaching methods on how students think about math and on their knowledge of the real world. However, they also stress that this student knowledge must be related to formal math concepts and notations. Furthermore, evidence indicates that understanding and skill do not develop separately but are continually intertwined and must be developed together. For example, it is more effective to intertwine student work on word and numerical problems than to do numerical problems separately first. The years of classroom research that underlie ***Math Expressions*** developed learning paths of supports and student strategies that can move students from their initial knowledge to fluency with and understanding of formal mathematical methods and notation.



BLENDING THE BEST OF TRADITIONAL AND REFORM

To simplify, in "traditional" classrooms teachers teach solution methods; in "reform" classrooms, students invent and discuss methods. ***Math Expressions*** does both. The research-based solution methods that are taught in the program help move students quickly to accurate and rapid-enough methods that are within the research-based learning path. In ***Math Expressions*** classrooms, both teachers and students demonstrate and explain. Students engage in practice activities to build fluency, but only after meaning-making activities in which initial understandings are built to guide the practice and increase understanding and skill interactively. ***Math Expressions*** students use activity sheets and do homework, but initial work has visual learning supports



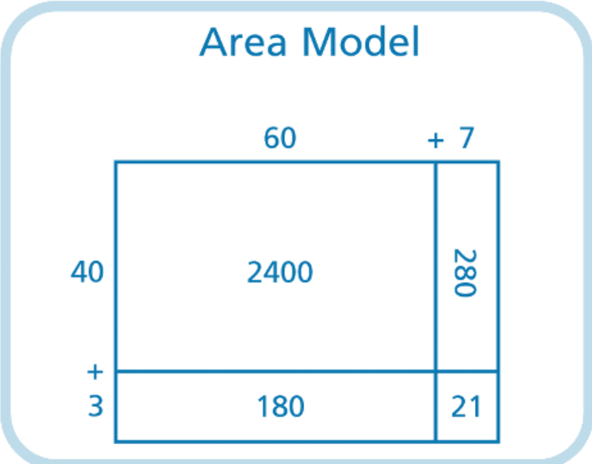
that help students link their initial knowledge to the formal math. Students make math drawings initially to help them build understanding and support their explanations of their solution methods. While homework is as close as possible to what parents have seen before, Math Letters, for example, describe things that are different and explain why the program uses these things. We ask each family to identify a "Home Helper" to support the child's learning from homework, as we have found that a strong home-school connection can facilitate the learning of all students. ***Math Expressions*** was developed in a range of different schools, including Spanish-speaking classrooms, so the student methods, linguistic and visual supports, and homework adapt well to families from many different backgrounds.

ACCESSIBLE ALGORITHMS AND MATH DRAWINGS

An algorithm is a multi-step method for solving a math problem. Traditional programs teach what many people think of as "standard" algorithms, i.e., "algorithms I learned when I was a student." Some reform programs introduced what have come to be called "alternative" algorithms. But the national reports and historical and international research are very clear that many different algorithms were used in this country during the past 150 years and many different algorithms are used around the world at the present time. So the issue is: Which algorithms make the most sense to teach now? Before calculators and computers, we needed many people who could calculate very efficiently, so complex algorithms were often taught. Now, people need to understand algorithms conceptually so they can actually comprehend the mathematics. Understanding and flexibility are important in today's mathematics. Some alternative algorithms used in reform programs do not relate

readily to the common algorithms, making discussion and understanding of these common algorithms difficult.

For all these reasons, ***Math Expressions*** has chosen research-based, student-friendly algorithms that are accessible to all students. They relate readily to the common algorithms, so parents and teachers can understand them but also see why the ***Math Expressions*** algorithms are more understandable by students. Students learn to make math drawings that show the quantities (e.g., drawing quick-ten sticks and circle ones for 67 + 43 or drawing a rectangle for 67 x 43). These help students build understanding of the math quantities and explain their method to other students by relating their numerical steps to steps in their drawing. Initially students may invent their own methods, but soon they also discuss the ***Math Expressions*** student-friendly algorithms, common algorithms, and any other algorithms that come into the classroom from homes.



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Math Expressions introduces two accessible algorithms (or two variations within one algorithm) for several reasons. All research found that certain students preferred different algorithms. Furthermore, having more than one algorithm allows students to discuss advantages and disadvantages of each one. The accessible algorithms were chosen to highlight vital math concepts so the discussion ensures that students engage with these concepts. Additionally, we found that such discussions did not confuse students and that this approach allows all students to find, use, understand, and explain an effective algorithm. Less-advanced students typically choose and use one algorithm, while other students may use and be able to explain more than one while becoming fluent with at least one. Students stop making math drawings whenever they can explain their numerical methods without drawings (they may still be asked to make math drawings when they explain their methods to other students). These visual supports and meaningful place-value language enable everyone to participate in Math Talk.

MATH TALK IN THE CLASSROOM

The national reports state how powerful it is for students to discuss their math thinking. But teachers have to build a classroom in which students feel safe discussing their thinking and their errors, and students need to see visual referents for the math solution methods being discussed in order to follow the discussion. ***Math Expressions*** provides support for teachers to build this classroom. The Teacher's Guides contain sample questions, explanations, and student-teacher dialogue to help teachers build a more advanced Math Talk classroom. ***Math Expressions*** introduces a Solve, Explain, Question, and Justify classroom structure in which students make math drawings at the board along with their numerical solution methods. Then, two or three students explain their methods while other students ask questions to stimulate more complete and adapted explanations. The teacher facilitates from the side or the back of the room to increase the amount of direct student-to-student dialogue. Initially, for any new math topic, the teacher may also need to model full explanations of some methods and help students explain more fully.

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Students also find and fix errors and explain why the errors are incorrect. This helps students overcome errors and prepares all students for the 21st century, in which mathematical understanding, debugging (finding and correcting errors), and verbal explaining are crucially important.

For all major grade-level topics, ***Math Expressions*** starts at the students' level and continually elicits their thinking, provides visual and linguistic supports to move them rapidly to understanding, and ends with extended fluency practice while continuing the emphasis on understanding and explaining. This takes more time than does traditional teaching. Therefore, the curriculum is organized into ambitious core grade-level topics that build over years to provide deep learning of the major core topics. Other topics are woven into these core topics: computation, word problems, and algebra are integrated; data topics fold into matching word problem types; geometry and measure are integrated and, in Grades 2–5, form mini-units that alternate with the core units and often build on those topics.

REFERENCES

Fuson, K. C. Developing Mathematical Power in Whole Number Operations. In J. Kilpatrick; W. G. Martin; and D. Schifter, eds. *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2003, pp. 68–94.

Fuson, K. C.; D. Wearne; J. Hiebert; P. Human; H. Murray; A. Olivier; T. Carpenter; and E. Fennema. "Children's Conceptual Structures for Multidigit Numbers at Work in Addition and Subtraction." *Journal for Research in Mathematics Education*, 28:130–162.

Fuson, K. C.; M. Kalchman; and J. D. Bransford. Mathematical Understanding: An Introduction. In M. S. Donovan and J. D. Bransford, eds. *How Students Learn: Mathematics in the Classroom.* Washington, D.C.: National Academy Press, 2005, pp. 217–256.

Fuson, K. C.; S. T. Smith; and A. Lo Cicero. "Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms." *Journal for Research in Mathematics Education*, 28:738–766.

Fuson, K. C.; Y. De La Cruz; S. Smith; A. Lo Cicero; K. Hudson; P. Ron; and R. Steeby. Blending the Best of the 20th Century to Achieve a Mathematics Equity Pedagogy in the 21st Century. In M. J. Burke and F. R. Curcio, eds. *Learning Mathematics for a New Century.* Reston, VA: National Council of Teachers of Mathematics, 2000, pp. 197–212.

Hufferd-Ackles, K.; K. C. Fuson; and M. G. Sherin. "Describing Levels and Components of a Math-Talk Community." *Journal for Research in Mathematics Education,* 35(2):81–116.

Kilpatrick, J.; J. Sowder; and B. Findell, eds. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press, 2001.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 2000.