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Math Expressions

# Building Concepts in *Math Expressions*

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*This is the first article describing the five major classroom structures found in the* **Math Expressions** *program. These structures are Building Concepts, Math Talk, Student Leaders, Quick Practice, and Helping Community. Though we discuss the five structures in separate author papers, they interact synergistically in the classroom.*

The Children's Math Worlds Research Project (CMW) *that developed the curriculum now called* **Math Expressions** *found that using these structures in the classroom enables children from all backgrounds to learn ambitious levels of mathematics with understanding, fluency, and confidence.*

## THE SENSE-MAKING CLASSROOM

Building concepts in the classroom requires experiences in which students use objects, drawings, conceptual language, and real-world situations, all of which help students build mathematical ideas that make sense to them. However, these meaning-building supports must be linked to formal mathematics notation, language, and methods so that the formal mathematics, too, becomes meaningful to students. Figure 1 shows how the sense-making classroom requires linking informal foundational mathematics to formal school mathematics through the use of mathematical language and drawings that require, but also help, students to reflect, abstract, and generalize.

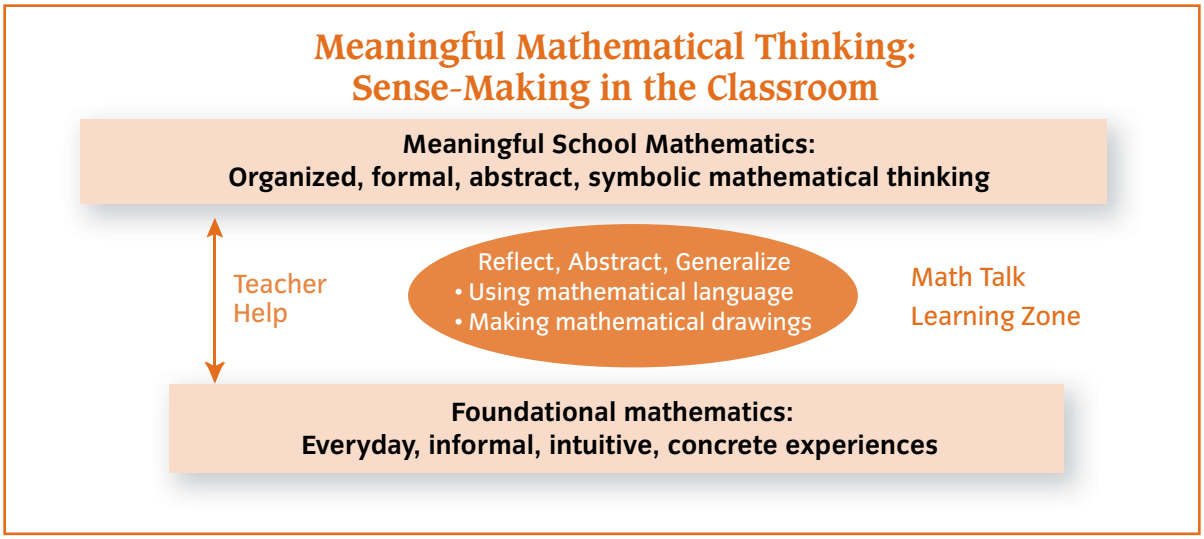
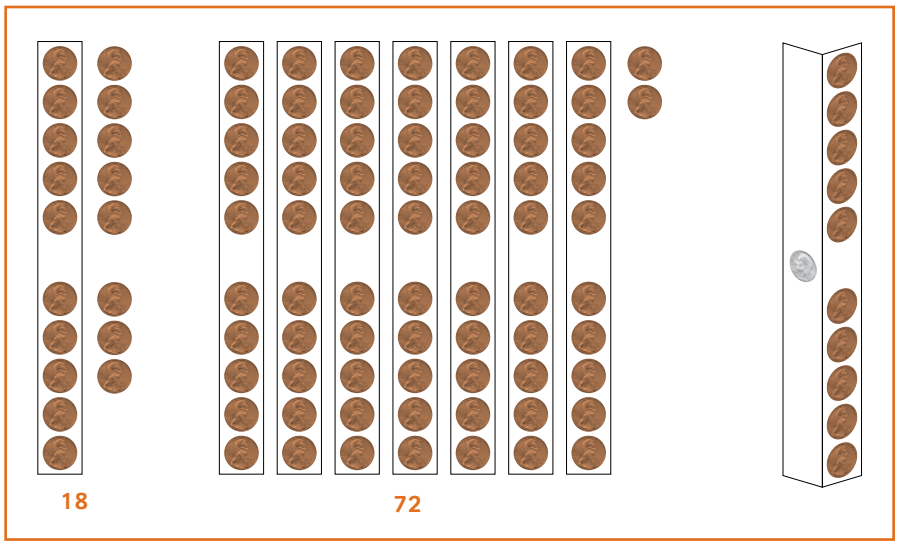



**Figure 1**

Students of all ages have experiences outside school that relate to mathematical concepts. These experiences can be brought into the classroom and discussed, role-played, drawn, and written about. ***Math Expressions*** encourages this and works to bring students' outside experiences into the classroom. We start with what students already know and work from there.

## CONCEPTUAL SUPPORTS

In our ten years of research with *CMW,* we found that many experiences outside school are not mathematically detailed enough to form the foundation for deep understanding. Instead, that foundation must be provided by experiences and learning supports within the classroom. For example, we found that many students in our classes had experiences buying and selling. Some of these experiences were in grocery and other stores, and some were with family members in various countries selling various products. However, many Kindergartners and Grade 1 students—and even some Grade 2 students—did not understand the quantities involved in money or even the concept of getting change from a purchase (i.e., that the change is less than the amount they pay). So we developed dime/penny strips with ten pennies in a column on one side and one dime on the other side, and nickel/penny strips with five pennies in a column on one side and one nickel on the other side. Buying and selling with these strips help students understand the values of coins and note how the smallest coin, the dime, is really of more value than the larger nickel coin (and that it is worth two times as much because two nickel strips make one dime strip). These strips are also useful in developing ideas of place value for numbers under 100 because any such number can be made with dime strips and nine pennies.




**Penny and Dime Strips**

## MEANINGFUL MATH DRAWINGS

***Math Expressions*** provides visual supports and word problems about real-world situations on class activity pages in the student workbooks. It differs from other programs in that it also provides a coherent sequence of research-based manipulatives, large dry-erase MathBoards with learning supports, math drawings for single-digit and multi-digit calculation and word problems, and conceptual flashcards that support understanding as well as fluency. The coherent learning paths for core math topics that build across grades are supported by these specially designed learning tools, which were developed during years of classroom research in many different classrooms with students from many different backgrounds.

Math drawings are a special focus of ***Math Expressions*** because they have so many advantages. They do not create management or behavior problems and do not get lost from year to year (or day to day) as do manipulatives. Making a drawing holds the attention of students with concentration or behavioral issues and is affirming to them as a mathematical product they can discuss. Drawings can be used on vertical chalkboards and on our MathBoards (which can then be shared with the class) so that the drawings support Math Talk. They can also be used for homework and class work so that teachers have a trace of student thinking they can look at after class. Students can make their own drawings for any mathematical situation, but we teach particular drawings for core math topics that relate to standard math notations and tools. Thus, the math drawings are a sense-making link between formal mathematics and informal sensory experiences from which the drawings were derived.

## MATH TALK LEARNING ZONE

Another difference from other programs is that ***Math Expressions*** systematically supports students engaging in Math Talk in the classroom, and the Teacher's Guides advise teachers on how to build such a classroom. Students from Grade 1 to Grade 5 move rapidly from manipulatives to math drawings (or proof drawings) that are related to their numerical solution methods. These math drawings enable everyone to follow the thinking of a given student and facilitate sense-making by all listeners. ***Math Expressions*** frequently uses a Solve, Explain, Question, and Justify classroom structure in which as many students as possible solve a problem at the board while others solve at their seats. Two or three students then explain their drawing and solution methods, and their classmates ask questions and discuss and expand the explanation.

Work with drawings is always preceded by work that builds up the drawings so that students understand how they show quantities or situations. Therefore, the drawings support sense-making for steps of numerical calculations, for structural aspects of word problem situations, or for geometric or data contexts. Thus, the Sense-Making Classroom (see Figure 1) that links informal and formal mathematics requires a Math Talk Learning Zone in which everyone builds meanings by using all their senses: seeing, hearing, speaking, body-sensing, and gesturing. Sharing such concept-building experiences helps the classroom community grow, and it creates a common mathematical language, both formal and informal, that facilitates student-to-student talk.

English language learners benefit from such a Math Talk Learning Zone with drawings and situations (and, earlier, manipulatives) because they can use all their senses to build meanings and learn and practice the English language. We have found through years of research that English language learners can learn to solve word

problems, even the ambitious kinds of word problems we use in ***Math Expressions*** that are similar to those given in other countries. English language learners just need many experiences solving, discussing, and writing word problems. Traditionally, many teachers and programs for English language learners have avoided word problems and writing. But word problems are small, mostly predictable texts that offer good opportunities for building language skills.

In the early research of *CMW,* we found that many Grade 2 and Grade 3 English language learners solved all word problems by adding the two numbers they saw; they did not even try to read the problem. With the *CMW* curriculum (now published as ***Math Expressions***), students learn a core word-problem solving strategy: read the problem, understand the situation, and make a drawing if it will help you. We have found that all English language learners improve greatly over the year because they have many opportunities to hear and speak English when there are sense-making supports in the classroom. Several years after our initial research in a school with students from Spanish-speaking backgrounds, we returned to the school and talked with a sixth-grade teacher of students who had previously been in one of our second-grade research classes. She said that she had never before seen students who could solve such word problems. They had enormous confidence and were not afraid to tackle any problem. They were still using the basic strategy they had learned: Read and understand the situation and make a drawing to help you.

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